ECON-UN 3211 - Running notes and updates

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Running log of points of emphasis and clarification as well as tracking updates to the slides made relative to what was presented in recitation. As usual, happy to rewrite bad penmanship, clarify slide content, or fix any lingering errors you guys spot. I try to make the annotated slides stand on their own as much as possible so also let me know if I can make them more so.

Logistics and math refresher

- My office hours have moved to in person *only* immediately after the recitation
- See 00-OptimizationReview.pdf for a quick refresher on calculus and optimization assumed for this course
- Recitation recordings
  - I intend to start uploading screen recordings of recitations the day the corresponding problem set is due (so usually the Thursday after the recitation)
  - Note I have not quality tested the audio, which may vary since I move around class
  - Since these are screen recordings, they will not include my blackboard notes, though I try to recreate these after class for the annotated slides I upload
- If I forget to upload my recitation slides or recordings on time, email me a kind reminder

1 Consumer Theory

1.1 The Utility-Maximization Problem

- For those who attended my recitation, a student pointed out after class a minor mistake in my derivation of the Cobb-Douglas demand functions that I sped through at the end. So to accurately state the result: the relative magnitudes of the exponents determine the share of income spent on the corresponding goods
- Prices do not affect these shares but they obviously affect the quantities these shares can afford so obviously income will enter into the demand function, contrary to what I said in class
- The annotated version of the slides are updated to reflect this

1.2 The Expenditure-Minimization Problem and special preferences

- As covered in recitation 3, there are two curves that trace the optimal bundle as income increases: the income offer curve does this in $x_2 - x_1$ space and the Engel curve does this in $m - x_i$ space for either good $i \in \{1, 2\}$
- In the original version of these slides, I mistakenly referred to the former as Engel curves in slides 11 and 24 on perfect complements and quasilinear preferences respectively. These are income-offer curves.

1.3 Comparative statics and the Slutsky decomposition of price effects

Discussion of income and substitution effects here pertain to the Slutsky method of decomposing a price effect on demand. Recitation 4 covers the Hicksian method, which will be favored for the rest of the course.

Most of the following bullet points are updates I mentioned in a course-wide email on October 1 after uploading the first version of annotated slides. These detail additions I made relative to the slides presented in recitation. However, please note the point on one additional adjustment, adding slide 28 on perfect substitutes.

- Slide 6: We talked about how when the demand for a good increases with income, it is a normal good. When it decreases with income, it is an inferior good. I just want to emphasize that this is a result of preferences: no good is 'inherently' a normal good or an inferior good. What I mean by that is:
  - The same good can be normal for someone with certain preferences but inferior for someone with different preferences.
  - Even for the same preferences, the same good can be normal for some intervals of income and inferior for other intervals of income. We touched on this possibility in last week's recitation and in the piecewise solutions in pset 3 when talking about quasilinear preferences.
  - Similarly, two goods can be substitutes for one set of preferences and complements for another
  - If we were to consider economies with more than two goods, these relationships get even more complicated. For example, one good may be a complement for another good but not vice versa. We will not consider these cases in this course, but just something to keep in mind.
- Slide 8: I skipped over luxury and necessary goods, two subcategories of normal goods, but the distinction between the two is whether or not demand for them increases more than proportionally to income increases (luxury good) or less (necessary goods). Equivalently, a luxury good is one whose demand elasticity of income is greater than 1 and a necessary

good is one whose demand elasticity of income is less than 1 (but still positive, otherwise it would be an inferior good).

- Slide 10: I added an additional Engel curve for the perfect substitutes case to depict the case when you consume good 1 and not good 2
- Slide 11: minor correction on the last bullet point: the denominator should be beta not alpha
- Slide 13-15: I cut our discussion on the comparative statics of price for time but I included most of what I wanted to say here
- There are price analogs for the income offer curve and the Engel curve from the comparative statics of income
- For own price, these are respectively called the price offer curve and, maybe obviously, the demand curve which we expect should slope downwards.
- Slide 15: I added a note here that is minor but possibly still helpful for thinking about income and substitution effects. Here, I consider perfect substitutes and perfect complements as, in a sense, the two 'extremes' for preferences. Most preferences are somewhere in between, containing features of both. The meaning becomes clearer later for slides 25-27.
- Slide 20: I said in class that the substitution effect is always nonnegative. A very important clarification here:
  - It is always non-negative when we're talking about a price decrease for the good whose price decreased (i.e. when purchasing power has increased)
  - It is always non-positive when we're talking about a price increase for the good whose price increased (i.e. when purchasing power has decreased)
  - The way to think about this is in the graph, a price decrease will make affordable bundles to the right of the original bundle, meaning they have more of the good whose price decreased. The opposite is true for a price increase.
  - These signs are opposite for the good whose price stayed the same.
  - We say non-negative/non-positive instead of positive/negative because some preferences will have zero substitution effect.
- Similar for slide 22: these relationships describe normal and inferior goods under a price decrease (green). The signs are flipped for a price increase (red) which is the case I've drawn alongside it.
- Slide 25-27: I added more complete graphs for perfect complements and substitutes
  - We see now what I meant by the note on slide 15: perfect substitutes only has a substitution effect and zero income effect while perfect complements only have an income effect and zero substitution effect. Most preferences have a combination of the two.
  - Helpful also to think about quasilinear preferences again. Recall Week 2 recitation notes or the Pset 3 on when it has zero income effect.
  - I added two depictions of perfect substitutes showing the case where the price change doesn't change which good we consume and then the case where it does change which good we consume. In both cases, there is zero income effect. (Update: see the first note)
- **UPDATE:** Slide 28: Important clarification on slides 25-27 on the Slutsky decomposition of a price effect for the case of perfect substitutes. Slides 25-27 looked at two cases:

  1. when a price change does not change which good is demanded
  2. when a change in the price of the good that is initially demanded changes which good is demanded

  Slide 26 says "there is always no income effect for perfect substitutes". This is true for the cases we looked at but what this is not a general property of perfect substitutes preferences.

- I've now added a slide 28 covering a third case that was omitted since it was covered in problem set 4: when a change in the price that is initially not demanded changes which good is demanded. In this third case, there is a non-zero income effect since $x(p', m) \neq x(p', m')$.

1.4 Hicksian decomposition of price change and consumer welfare

No notes

2 Producer Theory

2.1 The Cost Minimization Problem

- Slide 21: Added annotations on interpreting fixed-proportions (perfect complements) production in a way similar to the "compound good" in the consumer' sproblem
- Slide 22-23: Cleaned up figures for perfect substitutes production
- Slide 24: Elaborated on the Cobb-Douglas output elasticities interpretations showing how this relates to the returns to scale discussion in slide 19
- Slide 26: Added conditional factor demand functions for Cobb-Douglas (no working since it is tedious). Worth looking at these equations and imagining the comparative statics of factor demands: how does demand for the inputs change as the different parameters $(A, q, \alpha, \beta, w_1, w_2)$ change? Might

take some re-arranging but you can compare how this expression relates to the result from the consumer choice Cobb-Douglas case that budget share going to the two goods are invariant to price.

- Slide 27: A bit more messy: the Cobb-Douglas cost and marginal cost functions
- Slides 29-35: Added diagrams depicting different important relationships between the different cost functions

2.2 The Profit Maximization Problem and Supply Choice

Recitation 6 meant to cover the rest of producer theory. Unfortunately, it took place in the middle of midterm season and (understandably!) no one attended our optional Friday recitation. But for this reason, I do not have a recording for this material that I think is important. I have however uploaded annotated slides anyway. Let me know if anything here is unclear!

3 General equilibrium

3.1 The Edgeworth box

Unfortunately couldn't go over the Edgeworth slide material in recitation because the deadline for PS6 got extended to the day of recitation. So we prioritized going over common mistakes in the midterm and practice problems for PS6 and PS7 in class. Let me know if any of this material we didn't go over is unclear.
Some important adjustments here if anyone has been using or downloaded the version of the recitation slides uploaded on Friday (November 4, day of the rectitation) before being replaced on Saturday (November 5).
Basically, there's annoyingly little agreement among economists about the names assigned to the different components. My original slides and annotations correspond to Mas-Colell and Winston's convention. I have uploaded and added additional annotations to adjust to how this course teaches it. So updating my slide 22 and solutions to the practice problems by using writing in red to cross out the original names and replacing them with this class's names for those objects:

- Slide 22 (first figure): The curve tracing out where all indifference curves are just tangent to another
  - Original slide called this the Pareto set
  - New slide calls this the contract curve
  - Also appears in the practice problems on slide 36 and 37
- Slide 22 (second figure): The subset of the above curve that represents a Pareto improvement
  - Original slide called this the contract curve
  - New slide calls this the core
  - Also appears in the practice problem on slide 36
- Slide 22 (second figure): the area in green representing all Pareto improvements on the initial endowment
  - Original slide called this the core
  - New slide calls this the set of Pareto improvements

From Recitation 8:

- Slide 17 says "there is a unique budget line through the initial endowment that clears the market and its slope is orthogonal/perpendicular to the contract curve."
  - This is true except the part at the end about the slope being orthogonal/perpendicular to the contract curve. It doesn't change the validity of the approach, but this technically is not necessarily true.

4 Non-competitive Markets

- A student pointed out a minor mistake in my derivation of the solution to part a of Practice Problem 2 of my Recitation 9 slides. I've updated the slides but also posted the specific part that got corrected as 09-PracticeProblem2correction.pdf. It does not meaningfully change the rest of the solutions.
- Recitation 10, slide 10 on Cournot vs. Stackelberg. Important to understand what the difference is here between how to solve the two models
  1. Under Cournot, we take first-order conditions for both firms, re-arrange them into best-response functions, and then solve them simulataneously (or plug one into the other)
  2. Under Stackelberg, the best response function for the firm that moves second is still the same, but instead of solving them simultaneously or plugging one into the other, you plug that best-response function into firm 1's profit function and then derive a new first-order condition

Student questions

I get more emails than this, but just editing and selecting the ones that might have general relevance.

Problem Set 3: I'm having trouble finding the MRS in order to find Marshallian demand. I'm unsure whether the process is supposed to be very complicated or not but if it would be possible to have a little guidance I would really appreciate it!

The MRS is just the ratio of the marginal utility with respect to good 1 (MU1) to the marginal utility with respect to good 2

(MU2). In this case, there are a couple of ways to approach this that are equivalent to one another:

1. The first way is to apply the chain rule to the given utility function to get its partial derivatives with respect to each of the two goods, MU1 and MU2. See Section 3 of my Optimization Review notes for an example of the chain rule. In this case, the function $f(x) = x^2$ and $g(x) = x_1^{\frac{1}{2}} + x_2^{\frac{1}{2}}$.
   - When calculating MU1, differentiate $g(x)$ with respect to x_1 to get $g'(x_1)$
   - Obviously, $f'(x) = 2x$. Therefore, $f'(g(x)) = 2g(x)$
   - Therefore, $MU_1 = 2g(x) \times g'(x_1)$
   - Repeat with respect to x_2 to get MU_2, then $MRS := \frac{MU_1}{MU_2}$
2. A quicker way to approach this problem is to recall what I mentioned in recitation 1, that utility functions are invariant to monotonic transformations. This means, we can apply a monotonic transformation to $u(x)$ to get a new utility function $v(x)$ that is easier to work with that still has the same MRS and optimal demands.
   - Here, the best transformation is to define a new utility function $v(x) = \sqrt{u(x)} = x_1^{\frac{1}{2}} + x_2^{\frac{1}{2}}$
   - The MRS is then equivalent to the partial derivative of $v(x)$ with respect to x_1 divided by the partial derivative with respect to x_2, both of which don't require the chain rule to derive
   - You'll find the MRS still ends up exactly the same as in the first approach.

The second approach is easier as I said, but it's important to be comfortable using the different derivative rules (product rule, quotient rule, chain rule, etc.) as in the first approach in case you run into a utility function that isn't as easy to transform.

A related note on this front is that taking the log form of Cobb-Douglas functions is often easier to take derivatives of since it becomes additively separable compared to the exponent form.

Problem Set 3: A question on Question 3 on Shephard's Lemma: "I've gone over my calculations multiple times and I can't quite figure out what is going wrong that's not allowing me to simplify the partial derivative. Is it possible that my initial Hicksian demand is incorrect, or are the errors in my arithmetic or approach to the derivative?" [attached photo of working out]

You're doing the question absolutely perfectly, the algebraic simplification for this question is just surprisingly messy and unintuitive. I had trouble with it myself.

Best tip I can give is to keep comparing your work to the expression that you know you should get eventually. So if you put them side by side with the demand function on the LHS and the partial derivative on the RHS,

- You know that you want the RHS to eventually look like $\overline{u} \times p_1^{1-\alpha} \times p_2^{\alpha-1} \times \alpha^{1-\alpha} \times (1-\alpha)^{\alpha-1}$, which is a very clean multiplicative expression
- Once you do that, then you know that whatever's left inside the brackets, no matter how messy it looks as long as it's only a function of α and $(1-\alpha)$, must be reducible to $\frac{\alpha}{1-\alpha}^{1-\alpha}$

This last simplification might still take some work, but at least you know you're making progress the closer your expression looks to the LHS. If it's easier, you can also try it the other way around: trying to make the LHS look more like the RHS.

Problem Set 3: I attended your recitation session but am still confused about Question 1b on piecewise functions. Does this count as a piecewise function? [attached photo] Do I need to write the condition after "if" in terms of price? Or is what I have right now sufficient?

I have an example of piecewise functions in the annotated slides I uploaded earlier today [Recitation 2] using a perfect substitutes utility function as an example which I didn't have time to cover yesterday. See Slide 21. Your format is acceptable and I wouldn't take any points off as long as the solution is correct, but it would be more standard if you followed this format where it is

$$\begin{aligned} x_1(p_1,p_2,m) &= \begin{cases} \text{[value 1]} & \text{if [condition/case 1]} \\ \text{[value 2]} & \text{if [condition/case 2]} \\ \text{[value 3]} & \text{if [condition/case 3]} \end{cases} \\ x_2(p_1,p_2,m) &= \begin{cases} \text{[value 1]} & \text{if [condition/case 1]} \\ \text{[value 2]} & \text{if [condition/case 2]} \\ \text{[value 3]} & \text{if [condition/case 3]} \end{cases} \end{aligned} \tag{1}$$

or alternatively, as bundles:

$$x(p_1,p_2,m) = \begin{cases} (x_1 \text{ value 1, } x_2 \text{ value 1}) & \text{if [condition/case 1]} \\ (x_1 \text{ value 2, } x_2 \text{ value 2}) & \text{if [condition/case 2]} \\ (x_1 \text{ value 3, } x_2 \text{ value 3}) & \text{if [condition/case 3]} \end{cases} \tag{2}$$

The other thing I'd be mindful of is that because the question gives you a specific utility function, you should be able to express your answers in terms of parameters. In this example, because we want an expression for the demand functions $x(p,m)$, we want the values and the cases to be expressed only in terms of p_1, p_2, and m as in my example and not to have any functions of variables in the values or cases.

Similarly, if we were looking for Hicksian demand functions $x(p, \overline{u})$, we'd like the solutions to be expressed only in terms of p_1, p_2, and $\overline{u}$, no functions.

Finally, also be sure that your piecewise function covers all possible cases. So if you use my slide example for demand under perfect substitutes preferences, the piecewise function depends on how the price ratio $\frac{p_1}{p_2}$ compares to the threshold value $\frac{\alpha}{\beta}$. Thus, my piecewise function should include three cases:

1. when $\frac{p_1}{p_2} > \frac{\alpha}{\beta}$
2. when $\frac{p_1}{p_2} < \frac{\alpha}{\beta}$
3. when $\frac{p_1}{p_2} = \frac{\alpha}{\beta}$

A common mistake in the problem set was to forget the equality case!

Of course, it might be appropriate for a given problem to include the equality case by using weak inequalities so you might instead only have two cases to consider like:

1. when $\frac{p_1}{p_2} > \frac{\alpha}{\beta}$
2. when $\frac{p_1}{p_2} \leq \frac{\alpha}{\beta}$

Which approach is appropriate depends on the specific problem, but the point I'm making here is that you have to account for all possible cases for the piecewise to be properly defined.

Midterm

See start of Recitation 7 for some points of emphasis based on the questions that tripped students up the most

Problem Set 7 (Edgeworth boxes): I am looking at the sample question on the recitation slides. I am wondering how can we tell if the two indifference curves intersect in the box or outside of the box (like the picture you drew on the slides)? Or does it not matter?

That's a good question because I didn't show how to do that in my slides. For any set of preferences, you know that the relevant indifference curve must go through the endowment point. So if you calculate the level of utility at the endowment point, that must be that person's level of utility for all points on their indifference curve. So you can just plug that utility into the utility function to get the equation for the indifference curve. If you do this for both A and B, you can draw the indifference curves precisely and/or work out where it intersects the edges of the Edgeworth box. Alternatively, you can set them equal to one another (after converting units of A to units of B or vice versa) to find their points of intersection and see if it's inside the Edgeworth box.

(I posted a step-by-step elaboration of that part of my slides as 08-GraphingExample.pdf)